A New Fitness Evaluation Method Based on Fuzzy Logic in Multiobjective Evolutionary Algorithms

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***Abstract*—Evolutionary algorithms have been effectively used to solve multiobjective optimization problems with a small number of objectives, two or three in general. However, when encounter problems with many objectives (more than five), nearly all algorithms performs poorly because of loss of selection pressure in fitness evaluation solely based upon Pareto domination. In this paper, we introduce a new fitness evaluation mechanism to continuously differentiate solutions into different degrees of optimality beyond the classification of the original Pareto dominance. Here, the concept of fuzzy logic is adopted to define fuzzy-dominated relation. As a case study, the fuzzy concept is incorporated into the NSGA-II, instead of the original Pareto dominance principle. Experimental results show that the proposed method exhibits a better performance in both convergence and diversity than the original NSGA-II for solving many-objective optimization problems. More importantly, it enables a fast convergence process.**

***Index Terms*- Pareto optimality, multiobjective evolutionary algorithm, fuzzy logic, NSGA-II**

I. INTRODUCTION

Evolutionary algorithms have been effectively used to explore the Pareto-optimal front in multiobjective optimization problems (MOPs). In literature, most of these multiobjective evolutionary algorithms (MOEAs) and their variants based on evolutionary strategy, particle swarm optimization, differential evolution, or artificial immune system, work well only in problems with a small number of objectives, mainly in two or three dimensions. However, many real-world multiobjective optimization problems involve more than five conflicting objectives, which are commonly referred to as many-objective optimization problems and the performance of most MOEAs deteriorate severely in problems with such a large number of objectives [1].

The main reason that MOEAs lose the exploring capability in solving many-objective optimization problems is largely due to the *ineffective* definition in the Pareto Optimality. Consider the definition of Pareto-dominance relation.

Pareto Dominance:
For a minimization problem, a vector $u = f(x_u) = (u_1, \dots, u_k)$ is said to dominate $v = f(x_v) = (v_1, \dots, v_k)$, denoted by $u \prec v$, if and only if $\forall i \in \{1, \dots, k\}, u_i \leq v_i$ and $\exists i \in \{1, \dots, k\}, u_i < v_i$. ■

When any two given vectors u and v are compared, there are only two possible conclusions according to Pareto-dominance:

- Either u dominates v or v dominates u, or
- u and v are non-dominated with respect to each other.

Therefore, two vectors can only be differentiated under the first condition. However, from the definition, the more the number of objectives is, the harder the first condition can be satisfied. For example, consider u dominates v in a two-dimension MOP and a five-dimension MOP. To arrive at this relationship, in the two-dimension problem, u only need to be better than v in one objective and no worse in the other objective. However in the five-dimension problem, u should be better than v in at least one objective and no worse than v in the remaining objectives. On the other hand, it is fairly easy to arrive at "non-dominated with respect to each other" relation when two individuals are compared based on Pareto domination in a higher-dimensional problem.

Figure 1 shows the percentage of non-dominated individuals in the initial populations (i.e., set at 100, 200 and 500) randomly generated for the benchmark function DTLZ 2 [2], a scalable benchmark problem, under various number of objectives (i.e., from 2 to 50). Every data point is derived from 50 independent runs. From the figure, the proportion of non-dominated individuals rises quickly with the number of objectives increasing from two to five. When the number of objectives exceeds five, the proportion of non-dominated individuals in a randomly generated initial population is higher than 90%. This leads to no selective pressure during the evolutionary process no matter how MOEA is designed.

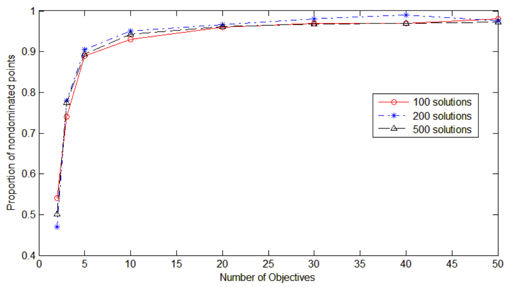

Figure 1: Percentage of non-dominated individuals

Therefore, although Pareto optimality is effective to ensure the convergence of the population in low dimension problems, it's nearly as ineffective maintaining selection pressure during evolution process in many-objective optimization problems.

In addition to the deficiency in the definition of Pareto dominance stated above, other complications such as visualization of high-dimension objective spaces [1], a large number of individuals needed to well represent the Pareto front [1], and a very high computational cost [3] have contributed to the challenges of solving many-objective optimization problems.

From the above discussions, all of the difficulties are caused by a large number of objectives. Naturally, the efforts in addressing this issue have led to strategies to reduce the number of objectives without losing information. For instance, Brockhoff and Zitzler [3] first identify conflict and non-conflict relationships between each pair of objectives and then combine non-conflict objectives into one objective. Deb and Saxena [4] propose a principle component analysis method to adaptively finding the correct lower-dimensional interactions by progressing iteratively from the interior of the search space towards the Pareto-optimal region. Singh *et al*. [5] identifies whether each objective is redundant or not based on an approximated non-dominated front which is roughly generated beforehand.

Although objective reduction works in some special conditions, there remain serious limitations. Of course, in the real-world environment, there exist problems whose objectives cannot be further reduced. In these problems, the methods stated above can only rely upon a relative order of importance of the objectives [4]. In some problems a very small number of objectives can be eliminated, but it may not make any difference. For example, in a ten-dimension problem, reducing only one or two objectives does not help much to solve the problem in an effective manner. Even if the number of objectives is reduced sufficiently, it is not clear how the Pareto front derived in the reduced low-dimensional space can portray the true Pareto front in the original high-dimension space.

Moreover, Said *et al*. [6] use the decision maker's preferences to set an error constant δ and incorporate it into Pareto-dominance to guide the search toward the area of interest in the Pareto front.

Because of the drawbacks in objective reduction methods and the lacking of preference information in most of the real-world applications, in this paper, our focus is placed solely on designing a new fitness measure through the definition of Pareto dominance to continuously differentiate individuals into different degrees of optimality beyond the classification of the original Pareto-dominance. Here, the notion of fuzzy logic [7] is adopted. Based on it, a fuzzy-dominated relation (FD) is defined and incorporated into the NSGA-II [8], as a case study, instead of the original Pareto dominance principle. The resulted fuzzy dominance NSGA-II is applied to search for Pareto optimal set in many-objective optimization problems by maintaining the selection pressure toward the Pareto front throughout the entire evolutionary process. Please note the same fuzzy dominance concept can be easily incorporated into other MOEA designs.

The remaining sections complete the presentation of this paper. Section II outlines selected approaches for fitness evaluations in literature. Section III elaborates the proposed fuzzy Pareto-dominated relation in detail and how to incorporate it into the NSGA-II. Section IV details the experiment setting and findings for two selected scalable benchmark problems. Finally, a conclusion is drawn in Section V along with pertinent observations.

II. LITERATURE REVIEW

In this section, first, the fitness assignment based on Pareto dominance principle in NSGA-II is briefly overviewed. Afterwards, some fitness evaluation approaches in literature are reviewed. Majority of existing multiobjective optimization algorithms exclusively uses the concept of Pareto domination. In these MOEAs, two solutions are compared on the basis of whether one dominates the other or not.

Pareto Optimality:
An individual $x \in \Omega$ is said to be Pareto optimal with respect to Ω if and only if there is no $x' \in \Omega$ for which $v = f(x')$ dominates $u = f(x)$. $f(x)$ is then called Pareto optimal (objective) vector. ■
Remarks: Any improvement in a Pareto optimal individual in one objective must lead to deterioration in at least one other objective.

Pareto Optimal Set:
The set of all the Pareto optimal individuals in the decision space is called Pareto optimal set (PS),
$PS = \{x \in \Omega \,|\nexists y \in \Omega, f(y) \prec f(x)\ \}$. ■

Pareto Front:
The image of the Pareto optimal set (PS) in the objective space is called Pareto front (PF), $PF = \{f(x)\ |x \in PS\}$. ■

In other words, a set of all Pareto optimal individuals form a tradeoff surface in the objective space. The basic definitions of dominance and Pareto optimality played an important role in the development of effective MOEAs. However, in MOP, Pareto domination does not define a complete ordering among the solutions in the objective space. Secondly, it does not measure that how much one solution is better than another one. A brief overview is given below to outline available researches for fitness evaluation in literature.

A. Existing Approaches for Fitness Evaluation

The first class of the existing approaches for fitness evaluation uses scalar methods instead of Pareto dominance to assign each individual a fitness value and compare them based on it. This class of designs can be further divided into four different categories. The first category using predefined weighting coefficients such as Weighted Sum (WS) [9] and Weighted Min-Max (Wmin-max) [10]. The second category focuses on extreme value of individuals, for example Maximum Ranking (MR) [9], Global Detriment (GD) [9], and Profit (PF) [9]. The third concerns about direct comparison of individuals including Favour Relation (FR) [9], k-dominance (KD) [11], and L-dominance (LD) [11]. The last category transforms objectives into constraints, for instance Ɛ-Constraint [10] and Goal Attainment (GAt) [10].

In the second class, suggested methods modify the Pareto-dominance to adapt it for the higher dimension problems, such as Pareto α-Dominance [12], Pareto ε-Dominance [12], and Pareto cone ε-Dominance [12]. Predefined parameters are incorporated into all of these methods. Each modified Pareto dominance design is a relaxing form of Pareto dominance in that it makes one individual dominates others easier in higher dimension optimization problems.

The third class is based on the idea of performance metrics. IBEA [13] is probably the most successful implementation of this class in that it has been shown to be more effective than other MOEAs in high dimension MOPs. There are other methods of this type: Volume Dominance (VD) [11] is based on the volume of dominated objective space by the individual, Contraction/Expansion of Dominated Area (CE) [11] adjusts the selection process by changing the size of individuals' dominance area and distance to the best known solution. GB [9] measures the best reference point's value that dominates the whole population.

Given the above discussions, researchers mainly focus on two different aspects in the fitness evaluation. First, Pareto dominance is replaced by another design, such as scalar method or performance metrics. Both approaches assign each approximation front an exact score and use this value for comparison. However, both of them only consider one specific characteristic of the front and the score generated cannot evaluate the front comprehensively. Second, Pareto dominance is modified by some predefined parameters. The choice of these parameters greatly affects the outcomes of fitness assignment. However, there exists no standard way or effective guideline to determine these parameters. In respond to these challenges encountered by the existing approaches, a new fitness evaluation measure based on fuzzy logic is proposed in this paper.

B. Previous Works in Fuzzy Dominance

In addition, there exist three known works following a similar research line.

Farina and Amato [14] designed the fuzzy-based definitions of optimality and dominated for human decision making in many-objectives optimization problems. The fuzzy definitions process preference information provided by the decision maker and generate a parameter whose value ranges from zero to one in order to compute different subsets of Pareto optimal solution set. When the value of the above parameter set to zero, the introduced definition is the same as classical Pareto-optimality. When the parameter value is increased, different subset of Pareto optimal solutions can be obtained corresponding to higher degrees of optimality.

In [15], a fuzzy Pareto dominance concept is introduced to compare two solutions and uses the scalar decomposition method of MOEA/D only when one of the solutions fails to dominate the other in terms of a fuzzy dominance level.

Köppen and Garcia [16] presented a generic ranking scheme that assigns fitness values, which represent the "dominating strength" of an individual against all other individuals in the population, to any set of vectors in a scale-independent manner. The fitness values reflect dominance degrees of vectors. Based on this ranking scheme, an extension of the Standard Genetic Algorithm, Fuzzy-Dominance-Driven GA (FDD-GA), was proposed.

III. PROPOSED METHOD

A. Background Knowledge of Fuzzy Logic

In this study, we define the membership function based on left Gaussian membership function (F_G)

[17], as shown in Figure 2 (in which $c = -1$ and $\sigma = 0.5$),

$$F_G(x) = \begin{cases} 1 & \text{if } x \le c \\ \exp\left(-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right) & \text{otherwise} \end{cases} \quad (1)$$

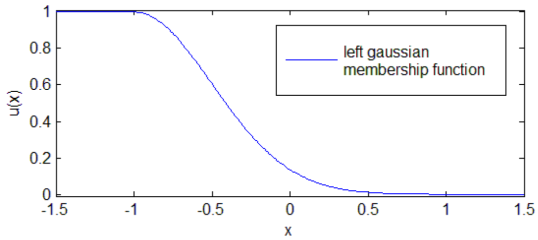

Figure 2: Left Gaussian membership function

This left Gaussian membership function is then applied in the definition of fuzzy-dominance relation. There exist several definitions in set theoretic operations which could be used to combine different fuzzy sets [17]. One definition uses "max" and "min" operations as follows:

- $u_{A\cup B}(x) = \max[u_A(x), u_B(x)]$
- $u_{A\cap B}(x) = \min[u_A(x), u_B(x)]$

where $u_A(x)$ is referring to the membership value of x under fuzzy membership function A. The second definition uses "*s-norm*" and "*t-norm*" as follows:

- *s-norm*: $x \oplus y = \min(1, x + y)$
- *t-norm*: $x \star y = \max(0, x + y - 1)$

The third definition is based on the product of fuzzy sets:

- $u_{A\cup B}(x) = u_A(x) + u_B(x) - u_A(x)u_B(x)$
- $u_{A\cap B}(x) = u_A(x)u_B(x)$

In the method proposed herein, according to the third definition, the individual's performance in each objective is considered as a fuzzy set and all these fuzzy sets are combined through product operator as the intersection of these fuzzy sets. Assume fuzzy set A describes how individual a is better than individual b in the first objective and fuzzy set B quantifies how individual a is better than individual b in the second objective. $A \cap B$ is the intersection of fuzzy sets A and B. It implies how individual a is better than individual b in both first and second objectives. The probability value of $A \cap B$ is $u_{A\cap B} = u_A \times u_B$. In fuzzy Pareto-dominance relation, comparison of two individuals in each objective is considered as a fuzzy set. Therefore, comparison of two individuals in all objectives is attained by the intersection of all fuzzy sets.

B. Fuzzy-dominance Relation

Without loss of generality, the definition of fuzzy-dominance relation is given in the following.

Fuzzy-Dominance Relaton:

In the k-objective minimization problem, given two individuals $u = f(x_u) = (u_1, \dots, u_k)$ and $v = f(x_v) = (v_1, \dots, v_k)$, define $p(u) = f(x_u) - f(x_v) = (u_1 - v_1, \dots, u_k - v_k)$ as the performance of u with respect to v and $p(v) = f(x_v) - f(x_u) = (v_1 - u_1, \dots, v_k - u_k)$ as the performance of v compared with u, respectively. The modified Gaussian membership function transforms each term of $p(u)$ and $p(v)$ into a probability measure in [0, 1]. That is $\varphi(u) = F_G(p(u)) = (\varphi_1^u, \dots, \varphi_k^u)$ and $\varphi(v) = F_G(p(v)) = (\varphi_1^v, \dots, \varphi_k^v)$. Here, in $\varphi(u)$, each $\varphi_i^u, i = 1, \dots, k$ is considered as a fuzzy set and set theoretic operators are applied to the product of all these fuzzy sets. Then we obtain a fuzzy product value of u, which is $\varphi_{\text{PRODUCT}}^u = \varphi_1^u \times \varphi_2^u \times \dots \times \varphi_k^u$. Similarly, fuzzy product value of v is defined accordingly as $\varphi_{\text{PRODUCT}}^v = \varphi_1^v \times \varphi_2^v \times \dots \times \varphi_k^v$.

Finally, compare $\varphi_{\text{PRODUCT}}^u$ and $\varphi_{\text{PRODUCT}}^v$, a vector u is said to fuzzy-dominate v (denoted by $u \prec_F v$) if and only if $\varphi_{\text{PRODUCT}}^u > \varphi_{\text{PRODUCT}}^v$. ■

If $\varphi_{\text{PRODUCT}}^u = \varphi_{\text{PRODUCT}}^v$, then u and v are fuzzy non-dominated with respect to each other. From the definition of φ_{PRODUCT}, if two individuals are fuzzy non-dominated with respect to each other, then both of them are also Pareto non-dominated with respect to each other. However, two individuals which are Pareto non-dominated with respect to each other do not necessary lead to that the same relationship in fuzzy non-domination. This characteristic makes fuzzy-dominance relation more effective than Pareto-dominance relation in many-objective optimization problems in which Pareto-dominance principle cannot preserve the needed selection pressure throughout the evolutionary process.

On the contrary, fuzzy-dominance relation can continue classifying high-dimensional individuals into different degrees of optimality. Therefore, fuzzy-dominance can be applied to assign fitness measures in the evolution process so as to overcome difficulties caused by the original Pareto dominance definition.

C. Fitness Assignment Based on Fuzzy-dominance

Assuming there are n individuals in the competition pool, every individual is paired with other $n - 1$ individuals respectively, forming $n - 1$ pairs of competitions. For individual a, in each of $n - 1$ pairs, it is compared with the other individual b using fuzzy-dominance relation and generates both fuzzy product values of $\varphi_{\text{PRODUCT}}^a$ and $\varphi_{\text{PRODUCT}}^b$. Then, fuzzy product value of a is normalized as $\varphi_{\text{PRODUCT}}^a / (\varphi_{\text{PRODUCT}}^a + \varphi_{\text{PRODUCT}}^b)$. This new normalized value is considered as the performance of a compared with b. After calculating each pair, add all performance values of a together to obtain a sum value. This sum value is then divided by $(n - 1)$ and the divided value is regarded as the fitness measure of individual a. Table I shows this fuzzy fitness assignment process.

TABLE I: Fuzzy fitness assignment process

```
Input:  a competition pool contains n individuals
FOR i = 1:n
    % calculate sum value of point i
    s(i) = 0;     % initialize sum value of point i
    FOR j = 1:n
      % calculate performance value of i with j, j ≠ i
      p = φ^i_PRODUCT/(φ^i_PRODUCT + φ^j_PRODUCT)
     s(i) = s(i) + p;
   END
END
Output: a n-dimension vector s with each dimension
          represents the sum value of an individual
```

D. Fuzzy-Dominance NSGA-II

In the original NSGA-II, two individuals are compared based on their rank values and crowding-distances. Pareto-dominance relation determines the rank value of each individual. In the proposed improved design, we will use fuzzy fitness assignment method instead, which is based on fuzzy-dominance relation to give each individual a rank value. After the rank value is determined, the same crowding-distance is used as the original design in NSGA-II.

TABLE II: The process of rank value assignment

```
Input:  a competition pool P contains n individuals
r=1;    % set initial rank value
% set the initial number of population that has been
   assigned rank value
c = 0;
% threshold value is heuristically chosen
a = 0.52;
IF c < n
  % first assign fitness value to each individual
  s=Fuzzy fitness assignment process;
  % fuzzy-non-dominated sorting
   FOR i = 1:(n − c)
        % if fuzzy fitness value of i is larger than a
           predefined threshold, it will be added to front r
        IF s(i) > a × (n − c)
           r_value(i) = r;     % assign rank to i
           c = c + 1;
           P = P − {i};
        END
    END
    r=r+1;
END
Output: rank values of n individuals
```

For every individual, if its sum value is larger than a predefined threshold, then place it in the first rank. After assigning rank value 1, remove all individuals in the first rank and consider the remaining ones. Do the same to assign rank value 2, rank value 3 and so on. One individual is fuzzy non-dominated with respect to others in the same rank. After all individuals have rank values assigned, calculate the crowding-distance for each individual. Table II explains how to assign the rank value to each individual and perform the fuzzy non-dominated sorting.

The NSGA-II is modified by fuzzy-dominance relations and the corresponding fuzzy fitness assignment. In order to fairly compare the performance of convergence by using fuzzy-dominance relations and Pareto-dominance relations, we use the same structure of the original NSGA-II. The only difference is that in the original NSGA-II, the fitness assignment is completed by Pareto-dominance, while in the modified NSGA-II (FNSGA-II), fuzzy-dominance principle is applied. Table III explains each step of the modified NSGA-II in detail.

E. Relation between Pareto Dominance and Fuzzy Pareto Dominance

TABLE III: The fuzzy-dominance NSGA-II

Input:	population size; the number of generations; fitness function; search space; Gaussian membership function;
Step1:	Initialize populations A random parent population P_0 is created. The population is sorted based on the fuzzy-dominance. Each solution is assigned a rank equal to its fuzzy-domination level where 1 is the best level. Binary tournament selection, recombination, and mutation operators are used to create a child population Q_0 of size N. Set $t = 0$
Step2:	$R_t = P_t \cup Q_t$ Combine parent and children population. The population R_t will have size $2N$.
Step3:	F=fuzzy-nondominated-sort (R_t) until $\|P_{t+1}\| < N$. PopulationR_t is sorted based on the non-domination sorting. The new parent population P_{t+1} is formed by adding solutions from the first front to the next best front before the size exceeds N.
Step4:	Calculate crowding distance-assignment (F_i) and include k-th non-dominated front in the parent population $P_{t+1} = P_{t+1} \cup F_i$
Step5:	Sort in descending order (P_{t+1}, $\geq_n$) and choose the first N elements of P_{t+1}: $P_{t+1} = P_{t+1}[0{:}N]$
Step6:	Q_{t+1}=make-new-pop (P_{t+1}) This population of size N is now used for selection, crossover, and mutation to create a new population.

In order to compare two different types of Pareto dominance, we first consider the difference between fuzzy set and crisp set. From [17], a fuzzy set is defined on a universe of discourse U (which provides the set of allowable values of a variable) by a membership function. The membership function measures how similar an element in U is to the given fuzzy set. The measurement result takes on value in a close interval, [0, 1]. Therefore, an element can belong to more than one fuzzy set with different degrees of similarity. For a fuzzy set A, $\mathrm{A} \cup \overline{\mathrm{A}} \neq$

U and $\mathrm{A} \cap \overline{\mathrm{A}} \neq \emptyset$. However, there does not exist one element belongs to two different crisp set. There are only two possible conditions between a crisp set U_c and an element e:

- $e \in U_c$ and $e \notin$ any other sets
- $e \notin U_c$ and $e \in$ one of any other sets

Obviously, fuzzy Pareto dominance is the relation between fuzzy sets while Pareto dominance only applies to the crisp sets. Therefore, for two individuals a and b, a fuzzy Pareto dominates b implies a partially Pareto dominates b or a Pareto dominates b with the probability in [0, 1]. When the possibility equals 1, a fuzzy Pareto dominates b implies a Pareto dominates b. When the possibility equals 0, a fuzzy Pareto dominates b means a is Pareto dominated by b. The concept linguistic variable is applied to describe this dominance degree. Linguistic variables are words and sentences in natural language describing the characteristic less specific than numerical ones [17]. Here, 'fuzzy Pareto dominance' is a linguistic variable; it can be decomposed into multiple terms based on different dominance degree (possibility):

- Pareto dominance (possibility = 1)
- Strong dominance (0.8 ≤ possibility < 1)
- Weak dominance (0.5 ≤ possibility < 0.8)
- Weak dominated (0.2 ≤ possibility < 0.5)
- Strong dominated (0 < possibility < 0.2)
- Pareto dominated (possibility = 0)

From the above discussion, Pareto dominance can be regarded as a special case of fuzzy Pareto dominance.

F. Comparison Between Our Method with Others

There exist some delicate distinctions between the proposed method and three other fuzzy-based approaches [14-16]. The distinctions can be roughly divided into three steps: fuzzification, inference and fuzzy rules, and defuzzification.

1) The proposed method

- Fuzzification
  In each objective, the single left Gaussian membership function is applied to transfer the difference between both solutions to a fuzzy value.
- Inference and Fuzzy Rules
  Product operation is applied to combine these fuzzy input values.
- Defuzzification
  Compare the fuzzy output with a heuristically chosen threshold, which is related to parameters of left Gaussian membership function used in fuzzification step.

2) Farina and Amato's Method [14]

- Fuzzification
  When compare a pair of solutions, all objectives of each solution are divided into three types: objectives better than, equal to, or worse than the other; each of which corresponds to left, center, and right Gaussian membership function, respectively.
- Inference and Fuzzy Rules
  t-norm operator is applied here.
- Defuzzification
  Compare the fuzzy output with a parameter generated by the definition of *K*-Optimality.

3) Method by Nasir *et al.*, [15]

- Fuzzification
  If one solution is better than the other in one objective, its membership function value of this objective is 1; otherwise, this value will be determined by the difference between two solutions in this objective through a pre-defined membership function.
- Inference and Fuzzy Rules
  Product operation is applied.
- Defuzzification
  Compare the fuzzy output with a domination threshold value which is related to decomposition method.

4) Köppen, Garcia and Nickolay's Method [16]

- Fuzzification
  When compare two solutions, in each objective, the minimum value between them is calculated as the membership function value in this objective
- Inference and Fuzzy Rules
  Product operation is applied.
- Defuzzification
  For each solution, all objective values of it are multiplied. Divide the production result obtained from the last step by multiplication of each solution, respectively. Compare these two results.

IV. EXPERIMENTAL RESULTS

A. Selected MOEAs for Comparison

In the experiment, three state-of-the-art MOEAs are chosen for comparison. They are the original NSGA-II [13], NSGA-II based on fuzzy-dominance relation (FNSGA-II), and SPEA 2 [18].

SPEA 2 [18] assigns a strength value to each individual in both main population and elitist archive which incorporates both dominated and density information. To avoid individuals dominated by the same archive members having identical fitness values, both dominating and dominated relationships are taken into account. The final rank value of an individual is assigned as the sum of the strengths of the individuals that dominate it. The density value of each individual is obtained by the nearest neighbor density estimation. The final fitness value is the sum of rank and density values.

B. Selected Benchmark Functions

Two widely used scalable many-objective benchmark problems, DTLZ2 and DTLZ3 [2], are chosen to evaluate the performance. In this experiment, chosen MOEAs are tested in three-dimension, five-dimension, and ten-dimension. From [19], the characteristics of these two test functions are with high-dimension objective space and multiple global optima.

C. Selected Performance Metrics

In this experiment, three performance metrics are chosen to quantify the performance. Generational Distance (GD) [20] measures the convergence of the Pareto front. Inverted Generational Distance (IGD) [21] considers both convergence and diversity at the same time, while Spacing [22] measures the distribution of individuals in the Pareto front. The less the GD, IGD, and Spacing values are, the better is the algorithm's performance.

D. Parameter Setting in Experiment

The population size in all three MOEAs is set to be 500 for all test instances. The stopping criterion is set at 250 generations. Initial populations are generated by uniformly randomly sampling from the search space in all the algorithms.

The simulated binary crossover (SBX) and polynomial mutation are used. The crossover operator generates one offspring, which is then modified by the mutation operator. Following the practice in [8], the distribution indexes in SBX and the polynomial mutation are set to be 20. The crossover rate is 1.00, while the mutation rate is $1/n$. and n is the number of decision variables.

E. Experiment Results

Detailed comparison results for 20 independent trials are presented in Tables IV-XI. In Tables IV-IX, each metric value is obtained from the mean value of 20 trials. Tables X-XI lists the mean time of 20 trials for each algorithm attaining a predefined GD value in both benchmark functions.

Tables IV-VI provide the experimental results for comparing the three chosen MOEAs in DTLZ2 given three-, five-, and ten-dimensional objectives.

TABLE IV: Performance metrics in 3-D DTLZ2

	IGD	GD	Spacing
NSGA-II	0.0927	0.0829	0.0120
FNSGA-II	0.0818	0.0778	0.0120
SPEA 2	0.1221	0.1113	0.0205

TABLE V: Performance metrics in 5-D DTLZ2

	IGD	GD	Spacing
NSGA-II	1.1492	1.9309	0.1798
FNSGA-II	0.7219	1.2967	0.0001
SPEA 2	1.2093	2.1092	0.0115

TABLE VI: Performance metrics in 10-D DTLZ2

	IGD	GD	Spacing
NSGA-II	1.0209	1.4092	0.0250
FNSGA-II	0.7532	0.9938	0.0001
SPEA 2	1.2124	1.5023	0.0119

Tables VII-XI present the experiment results for comparing the three chosen MOEAs in DTLZ3 given three-, five-, and ten-dimensional objectives.

TABLE VII: Performance metrics in 3-D DTLZ3

	IGD	GD	Spacing
NSGA-II	3.1409	14.7021	1.0091
FNSGA-II	2.7201	12.0692	0.8238
SPEA 2	3.1965	16.0962	0.9012

TABLE VIII: Performance metrics in 5-D DTLZ3

	IGD	GD	Spacing
NSGA-II	5.3024	19.9821	1.5329
FNSGA-II	5.2612	14.9503	1.1439
SPEA 2	13.3097	18.0254	3.3784

TABLE IX: Performance metrics in 10-D DTLZ3

	IGD	GD	Spacing
NSGA-II	18.9419	24.8443	5.0279
FNSGA-II	6.6543	16.0925	0.8348
SPEA 2	19.3178	26.0912	2.3176

From above results, in all test instances, NSGA-II based on fuzzy-dominance principle performs better than the other two algorithms. While the number of objectives increases, the improved performance of fuzzy NSGA-II based on fuzzy-dominance is growing appreciably.

In the following experiment, we are interested in how much improvement can be attained in term of convergence time for the NSGA-II based on fuzzy-dominance relation. The problem is posted as: if in five- and ten-dimension DTLZ2 problem, after 250 generations, the values of GD in NSGA-II are 1 and 1.5, respectively, how many generations are needed for FNSGA-II to reach the same value in GD? The same experiment is repeated on DTLZ3 as well. Here, in five- and ten-dimension DTLZ3 problem, after 250 generations, the values of GD in NSGA-II are 20 and 25, respectively. Tables X-XI show the experiment result in DTLZ2 and DTLZ3.

TABLE X: Comparison result in DTLZ 2

	5-D (GD=1)	10-D (GD=1.5)
NSGA-II	237	229
FNSGA-II	107	62

TABLE XI: Comparison result in DTLZ 3

	5-D (GD=20)	10-D (GD=25)
NSGA-II	249	243
FNSGA-II	161	103

The above experiment results support our assumption that NSGA-II based on fuzzy-dominance provides a faster convergence performance than that of the original NSGA-II.

V. CONCLUSION

From the experimental results, given the same test problem, NSGA-II based on fuzzy-dominance ensures a better performance in both convergence and diversity. Moreover, its evolution process is fast. The performance improvement from Pareto-dominance relations to fuzzy-dominance relations is clearly appreciable. Continuing research will be extended to other state-of-the-art designs in MOEA.

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